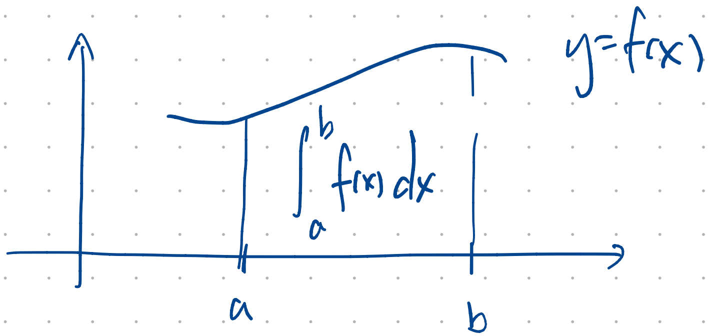
PREVIEW OF CH. 15

First, let's revisit a bit of SVC.

You learned e.g. how to compute an area under a curve:

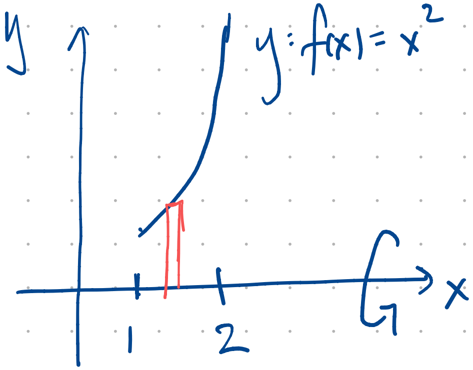


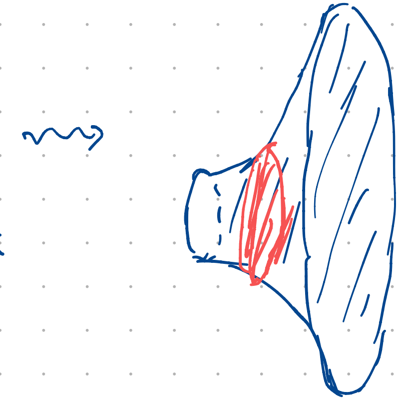
Visually:

"$f(x)\,dx$"

Add up areas of thin rectangles.

You also learned about solids of revolution:



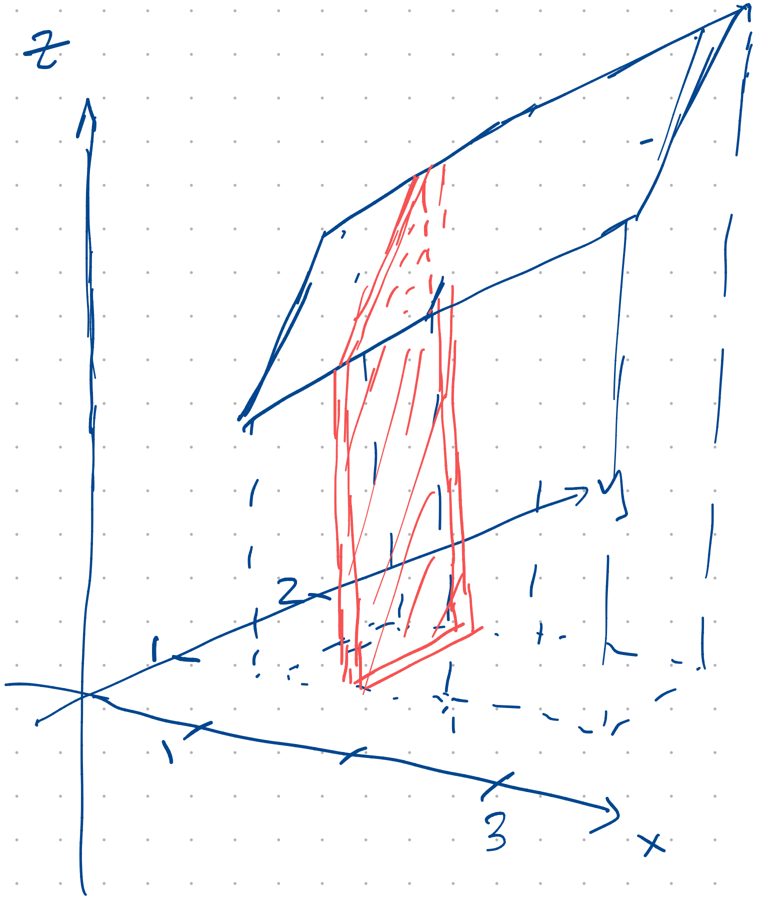


One way to find the volume of such a thing...

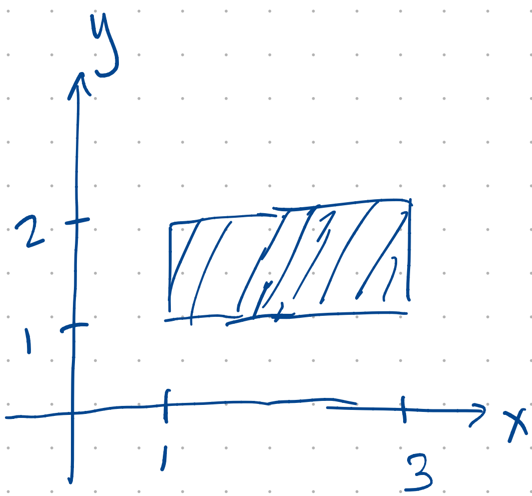
"$\pi f(x)^2\,dx$"

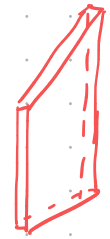
"Add" them up: $\int_1^2 \pi (x^2)^2\,dx$

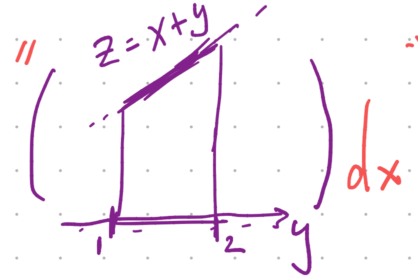
Observe: the sizes of the slices vary with x, i.e. is a function of x.



View from above:



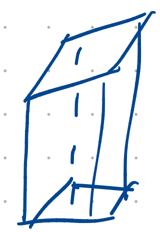




$$\int_1^2 (x+y)\,dy$$

constant in this slice!!!

So the volume of "is" $\left(\int_1^2 (x+y)\,dy\right)dx$

So the volume of  is the "sum"

outer integral adds up slices

$$\int_1^3 \left(\underbrace{\int_1^2 (x+y)\,dy}_{\text{area of each slice}} \right) dx$$

Let's compute:

$$\int_1^2 (x+y)\,dy = \left(xy + \frac{1}{2}y^2\right)\Big|_{y=1}^{y=2}$$

$$= (2x+2) - \left(x + \frac{1}{2}\right)$$

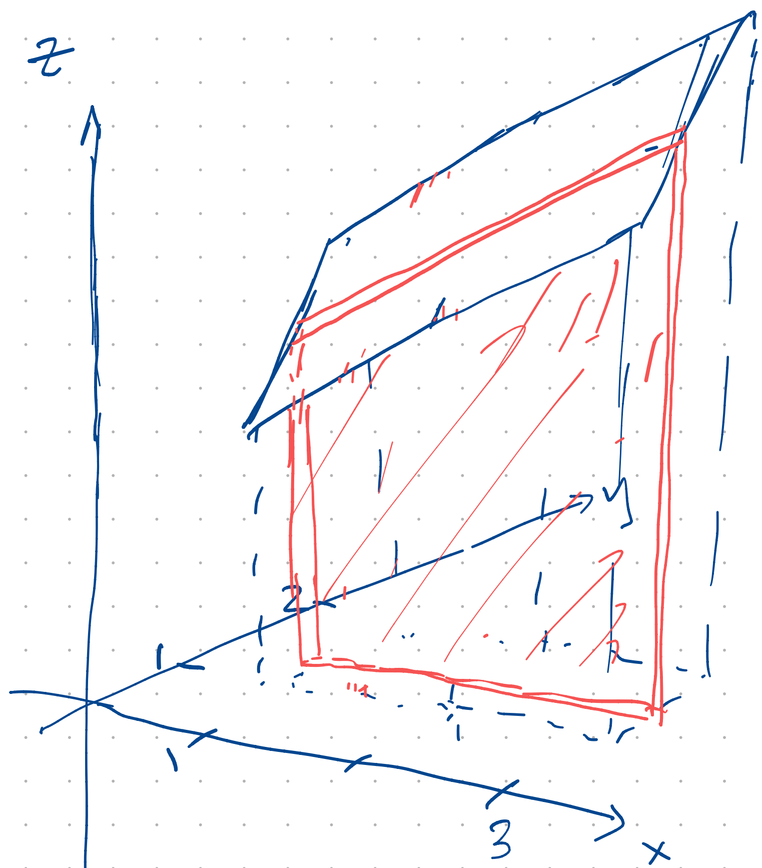
$$= x + \frac{3}{2}$$

Observe: this is a function <u>(only)</u> of x.

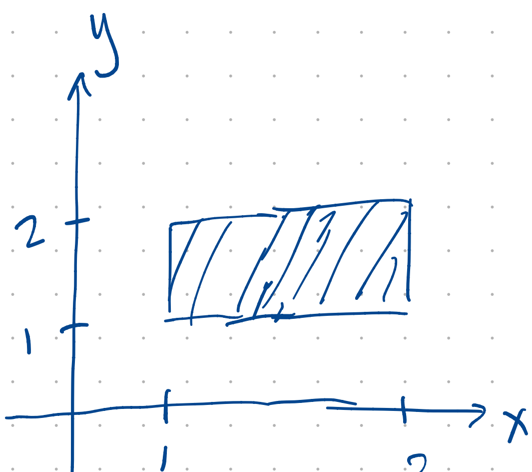
$$\int_1^3 \int_1^2 (x+y)\,dy\,dx = \int_1^3 \left(x + \frac{3}{2}\right) dx$$

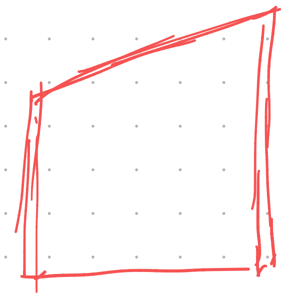
$$= \left(\frac{1}{2}x^2 + \frac{3}{2}x\right)\Big|_{x=1}^{3}$$

$$= \left(\frac{9}{2} + \frac{9}{2}\right) - \left(\frac{1}{2} + \frac{3}{2}\right) = \boxed{7}$$

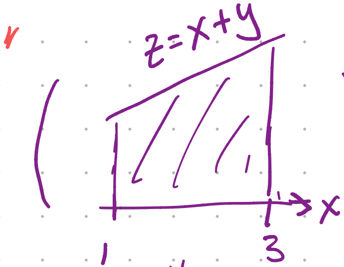


View from above:





$=$



dy

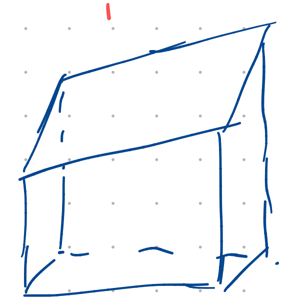
$\|$

$$\int_1^3 (x+y)\,dx$$

constant in this slice!!!

So this has vol: $\left(\int_1^3 (x+y)\,dx\right) dy$

Add them up:



$$\int_1^2 \left(\int_1^3 (x+y)\,dx\right) dy$$

COMPARISON

$$\int_1^3 \int_1^2 (x+y)\,dy\,dx$$

$= \cdots\cdots$

$= 7$

What you witness here $(7=7)$ is a consequence of Fubini's Thm (15.1.10).

Fubini is to integration as Clairaut is to differentiation.

$$\int_1^2 \int_1^3 (x+y)\,dx\,dy$$

$$= \int_1^2 \left(\tfrac{1}{2}x^2 + xy\right)\Big|_{x=1}^{3} dy$$

$$= \int_1^2 \left(\tfrac{9}{2} + 3y\right) - \left(\tfrac{1}{2} + y\right) dy$$

$$= \int_1^2 (4 + 2y)\,dy$$

$$= \left(4y + y^2\right)\Big|_{y=1}^{2} = (8+4) - (4+1)$$

$$= 7.$$

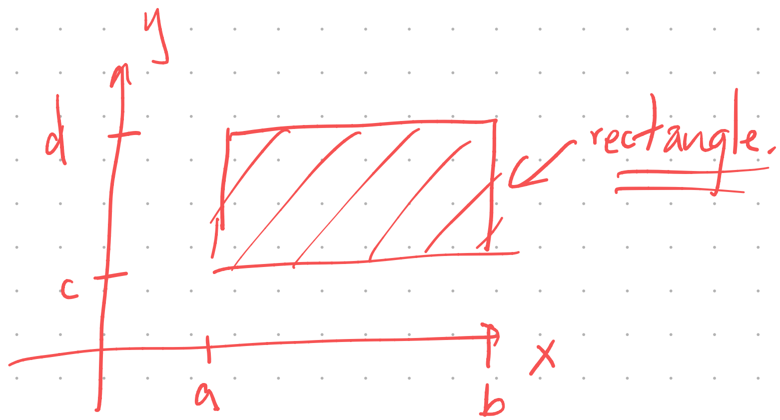
Rmk. There is no notion of an "antiderivative" of e.g. $f(x,y) = x+y$.

(The analogous notion is a "potential fn" for a "conservative vector field", but that's Ch. 16)

⚠ $$\int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

only works when a,b,c,d are constants,

i.e.



In general, can still do dydx or dxdy, but bounds will be harder to convert

(§15.2, discuss next time)